

Teaching program

 New Century Maths 7 for
the Australian Curriculum

Year 7 topics

Week	Semester 1
Term 1	
1	1 Integers (Number and Algebra)
2	
3	
4	2 Angles (Measurement and Geometry)
5	
6	
7	3 Whole numbers (Number and Algebra)
8	
9	
10	<i>Lost time</i>
Term 2	
1	4 Fractions and percentages (Number and Algebra)
2	
3	
4	5 Algebra and equations (Number and Algebra)
5	
6	
7	6 Geometry (Measurement and Geometry)
8	
9	
10	<i>Lost time</i>

Week	Semester 2
Term 3	
1	7 Decimals (Number and Algebra)
2	
3	
4	8 Area and volume (Measurement and Geometry)
5	
6	
7	9 The number plane (Number and Algebra, Measurement and Geometry)
8	
9	
10	<i>Lost time</i>
Term 4	
1	10 Analysing data (Statistics and Probability)
2	
3	
4	11 Probability (Statistics and Probability)
5	
6	
7	12 Ratios, rates and time (Number and Algebra, Measurement and Geometry)
8	
9	
10	<i>Lost time</i>

Curriculum strands

Number and Algebra

Measurement and Geometry

Statistics and Probability

Year 8 topics

Week	Semester 1
Term 1	
1	1 Pythagoras' theorem
2	(Measurement and geometry)
3	
4	2 Working with numbers
5	(Number and Algebra)
6	
7	3 Algebra
8	(Number and Algebra)
9	
10	<i>Lost time</i>
Term 2	
1	4 Geometry
2	(Measurement and Geometry)
3	
4	5 Area and volume
5	(Measurement and Geometry)
6	
7	6 Fractions and percentages
8	(Number and Algebra)
9	
10	<i>Lost time</i>

Week	Semester 2
Term 3	
1	7 Investigating data
2	(Statistics and Probability)
3	
4	8 Congruent figures
5	(Measurement and Geometry)
6	9 Probability
7	(Statistics and Probability)
8	
9	<i>Lost time</i>
10	
Term 4	
1	10 Equations
2	(Number and Algebra)
3	
4	11 Ratios, rates and time
5	(Number and Algebra, Measurement and Geometry)
6	
7	12 Graphing linear equations
8	(Number and Algebra)
9	
10	<i>Lost time</i>

Curriculum strands

Number and Algebra

Measurement and Geometry

Statistics and Probability

1 Integers

Time: 3 weeks (Term 1, Weeks 1–3)

Text: *New Century Maths 7*, Chapter 1, page 2

NSW/Australian Curriculum references: Number and Algebra

Whole Numbers 2/Number and place value

- Investigate everyday situations that use integers. Locate and represent these numbers on a number line (6NA124)

Computation with Integers/Number and place value

- Compare, order, add and subtract integers (7NA280)
- Carry out the four operations with integers, using efficient mental and written strategies and appropriate digital technologies (8NA183)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-3WM** recognises and explains mathematical relationships using reasoning
- **MA4-4NA** compares, orders and calculates with integers, applying a range of strategies to aid computation

Introduction

This Number topic introduces formal operations with integers. Students have already been introduced to the idea of numbers less than zero in primary school but, in this topic, they examine the four arithmetic operations with integers, so spend considerable time on the introduction and practice of such skills. Although the Australian Curriculum regards the multiplication and division of integers as being Year 8 content, we have included them here (and in *New Century Maths 8*) for consistency and completeness, and due to the need to use them in the ‘Algebra and equations’ topic.

Content

- 1 Numbers above and below zero** 6NA124 U F P S C
 - Understand that arithmetic laws provide powerful ways of describing and simplifying calculations and that using these laws leads to the generality of algebra
- 2 Integers on a number line** 6NA124 U F C
- 3 Ordering integers** 7NA280 U F C
- 4 Adding integers** 7NA280 U F R C
- 5 Subtracting integers** 7NA280 U F R C
- 6 Multiplying integers** 8NA183 U F R C
- 7 Dividing integers** 8NA183 U F R C
- 8 Order of operations** 8NA183 U F C
- 9 Integers and the calculator** 8NA183 F C
- 10 Integer problems** 7NA280 F P S C
- 11 Revision and mixed problems**

Related topics

Year 7: Whole numbers, Fractions and percentages, Algebra and equations, Decimals, The number plane

Year 8: Working with numbers, Algebra, Equations, Graphing linear equations

Proficiency strands/Working mathematically

- **U = Understanding (knowing and relating maths):** Understanding the concept of integers and operations on them
- **F = Fluency (applying maths):** Selecting appropriate integer operations, including the use of the calculator
- **PS = Problem solving (modelling and investigating with maths):** Using integers to solve real-life problems
- **R = Reasoning (generalising and proving with maths):** Extending the rules for operating with positive numbers to negative numbers, looking for general patterns in operations with integers (for example, $-4 + 7 = 7 - 4$)
- **C = Communicating (describing and representing maths):** Interpreting and writing integers

Extension ideas

- Investigate powers of negative numbers, for example, $(-1)^4$. See 'Power Plus' on page 32.
- Research the history of negative numbers, zero and Brahmagupta, or the set of integers J .

Teaching notes and ideas

- *Resources:* number line, spreadsheet
- Students were introduced to the concept of negative numbers in primary school, namely their position and order, but not formal operations with them.
- Students should also have had experience in Year 6 with positive number lines, timelines and order of operations.
- Introduce directed numbers using the idea of opposites, for example north/south, profit/loss, temperatures above and below zero, AD/BC (now known as CE/BCE), and time before/after take-off. Extend the number line backwards in order to answer questions such as: $4 - 9 = ?$
- Brahmagupta was a famous Indian mathematician who lived from 598 to 670 CE. He wrote important works about mathematics and astronomy, and is considered as the discoverer of the number zero. He defined zero as the result of subtracting a number from itself, and developed calculation rules for positive and negative integers, which he called 'fortunes' and 'debts' respectively.
- Applications of negative numbers: bank balance, temperature, profit and loss, elevators, indoor cricket scores, golf scores, T minus 5 seconds (5 seconds before take-off)
- In addition and subtraction, a negative number implies the opposite operation, that is $+(-b) = -b$, and $-(-b) = +b$. Adding a 'drop' of 8 means subtracting 8. Another way of thinking about $3 - (-4)$ is to count how many places there are between -4 and 3 on a number line.
- Discover the rules for multiplication through repeated addition.

Assessment ideas

- Writing activities: for example explaining why subtracting a negative number is the same as adding its opposite
- Open-ended questions: for example finding two negative numbers whose difference is 4
- Have groups of students write a story involving negative numbers or give a presentation to the class

Technology

Investigate on the calculator the difference between $(-3)^2$ and -3^2 . How are negative numbers represented on spreadsheets?

Language

- -3 is read as *negative 3*, not *minus 3*. Students should not confuse the negative sign with the sign for the minus operation (this is also true on a calculator). Once, negative numbers were written with the ‘ $-$ ’ sign superscripted; that is -3 instead of -3 .
- The NSW syllabus recommends this format for writing integers: $-2 - 3$, $-7 + (-4)$, $-2 - (-3)$, and says, ‘Complex recording formats for directed numbers such as raised signs can be confusing.’
- *Integers* refers to positive and negative *whole* numbers and zero, while the more general term for all positive and negative numbers is *directed numbers*.
- CE means *Common Era*, BCE means *Before the Common Era*, AD means *Anno Domini* (‘in the year of our Lord’), BC means *before Christ*.

2 Angles

Time: 3 weeks (Term 1, Weeks 4–6)

Text: *New Century Maths 7*, Chapter 2, page 36

NSW/Australian Curriculum references: Measurement and Geometry

Two-dimensional Space 2/Geometric reasoning

- Investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles. (6MG141)
- Use the language, notation and conventions of geometry. Recognise the geometric properties of angles at a point. (NSW Stage 4)

Angle Relationships/Geometric reasoning

- Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (7MG163)
- Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning (7MG164)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-3WM** recognises and explains mathematical relationships using reasoning
- **MA4-18MG** identifies and uses angle relationships, including those related to transversals on sets of parallel lines

Introduction

In this topic, students become familiar with the terminology and notation associated with angles and lines, building on elementary concepts introduced in primary school. The topic also marks the start of deductive geometry, as students are introduced to the properties of angles, including alternate, corresponding and co-interior angles on parallel lines. Please note that the Australian Curriculum considers complementary and supplementary angles, angles at a point, and vertically opposite angles as being Year 6 content while the NSW syllabus considers them to be Year 7 concepts.

Content

- 1 Naming angles** 6MG141 U C
 - Use the two alternative conventions for naming angles
- 2 Measuring and drawing angles** 6MG141 U
 - Measure, estimate and compare angles in degrees
- 3 Classifying angles** 6MG141, 7MG163 U C
 - Classify angles according to their sizes
 - Define and classify angles as acute, right, obtuse, straight, reflex or revolution, and pairs of angles as complementary, supplementary, adjacent or vertically opposite
- 4 Complementary and supplementary angles** 6MG141 U F C
- 5 Angles at a point and vertically opposite angles** 6MG141 U F C

6 Constructing parallel and perpendicular lines 7MG163 U C

- Construct parallel and perpendicular lines using their properties, a pair of compasses and a ruler, and dynamic geometry software

7 Corresponding angles on parallel lines 7MG163 U F C**8 Alternate angles on parallel lines** 7MG163 U F C**9 Co-interior angles on parallel lines** 7MG163 U F C**10 Angles on parallel lines** 7MG163 U F PS R C**11 Proving parallel lines** 7MG164 U F PS R C**12 Revision and mixed problems**

Related topics

Year 7: Geometry

Year 8: Pythagoras' theorem, Geometry, Congruent figures

Proficiency strands/Working mathematically

- **U = Understanding (knowing and relating maths):** Using the language, notation and conventions of angle geometry
- **F = Fluency (applying maths):** Using appropriate notation, rules and properties to solve geometrical problems
- **PS = Problem solving (modelling and investigating with maths):** Finding unknown angles in geometry problems
- **R = Reasoning (generalising and proving with maths):** Proving properties involving angles and parallel lines
- **C = Communicating (describing and representing maths):** Using angle notation and terminology

Extension ideas

- Investigate the history of geometry, Euclid, angle measurement and the Sumerian base 60 system.
- Use geometrical instruments to bisect angles and intervals.
- Investigate the angle sums of triangles and/or quadrilaterals (covered later this year in the 'Geometry' topic).

Teaching notes and ideas

- *Resources:* protractor (180° or 360°), ruler, set squares, compasses, paper and scissors, charts and posters, geostrips or Meccano strips, clock or whiteboard compasses (arms form an angle), navigational compass
- An angle can be demonstrated using geostrips, whiteboard compasses or even two strips of cardboard fastened together. Use terms such as *quarter-turn*, *half-turn*, *complete turn* first. In skateboarding jargon, what is a *360* or a *180*?
- Angles should be shown in different orientations, not all with horizontal 'bases'. Mark the angle that is being discussed, otherwise you could be referring to the larger reflex angle.

- According to the *Guinness Book of Records*, the steepest street in the world is Baldwin Street in Dunedin, New Zealand. The top of the street has a gradient of 1 in 2.86, which is an angle of 19° to the horizontal. Photos and videos showing the steepness of Baldwin Street can be found on the Internet.
- Organise an orienteering activity to demonstrate the use of angles in compass bearings.
- Use scissors, chopsticks, Meccano strips or geostrips to demonstrate properties of vertically opposite angles, and corresponding, alternate and co-interior angles on parallel lines.
- Draw perpendicular lines in different orientations, so that students don't confuse *perpendicular* with *vertical*.
- The sizes of alternate and co-interior angles may be deduced. Draw two parallel lines cut by a transversal, give the size of one angle, and ask students to find the size of the other seven and note any patterns.
- In their working, encourage students to write, for example, 'alternate angles *on parallel lines*' and not just 'alternate angles'. This is because alternate angles exist even when a transversal crosses lines that are not parallel. However, in those cases the alternate angles are not equal.

Assessment ideas

- Design a poster or present a class talk illustrating one or more of the angle properties
- Research the history of angle measurement
- The meaning of the geometrical term 'converse', the symbols \therefore and \because , and using letters of the Greek alphabet to label angles

Technology

There is much scope in this topic to use dynamic geometry software such as GeoGebra. The Internet is full of dynamic geometry animations that demonstrate the angle results shown in this topic.

Language

- *Acute* is from the Latin *acutus* meaning 'sharp' while *obtuse* is from the Latin *obtusus*, meaning 'blunt'.
- *Supplementary angles* refers to any two angles that have a sum of 180° , not necessarily two angles on a straight line so, when students are giving reasons in proofs, encourage them not to write 'supplementary angles' if they really mean 'angles on a straight line'. For example, co-interior angles on parallel lines are also supplementary. Similarly, *complementary angles* refers to any two angles that have a sum of 90° , not necessarily two angles that combine to make a right angle.
- From the NSW syllabus, under 'Angles': 'Students are to be encouraged to give reasons when finding unknown angles. For some students, formal setting out could be introduced. For example, $\angle ABQ = 70^\circ$ (corresponding angles, $AC \parallel PR$).' Note that the parallel lines must be mentioned.

3 Whole numbers

Time: 3 weeks (Term 1, Weeks 7–9)

Text: *New Century Maths 7*, Chapter 3, page 88

NSW/Australian Curriculum references: Number and Algebra

Whole Numbers 1/Number and place value

- Identify and describe factors and multiples of whole numbers and use them to solve problems (5NA098)

Addition and Subtraction 1/Number and place value

- Use estimation and rounding to check the reasonableness of answers to calculations (5NA099)

Whole Numbers 2/Number and place value

- Identify and describe properties of prime, composite, square and triangular numbers (6NA122)

Multiplication and Division 2/Number and place value

- Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (6NA123)

Indices/Number and place value

- Investigate index notation and represent whole numbers as products of powers of prime numbers (7NA149)
- **Determine and apply tests of divisibility (NSW Stage 4)**
- Investigate and use square roots of perfect square numbers (7NA150)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-3WM** recognises and explains mathematical relationships using reasoning
- **MA4-4NA** compares, orders and calculates with integers, applying a range of strategies to aid computation

Introduction

This topic revises mental and written operations with whole numbers before covering special number concepts, such as powers, roots, prime numbers, factor trees, greatest common divisor (highest common factor) and lowest common multiple. Analysing properties of numbers leads to the study of pattern and early algebra work. Less able students may have difficulty with the more abstract concepts in this topic, but factors and square numbers should have already been introduced in primary school. Please note that the Australian Curriculum considers the divisibility tests as being Years 5–6 content, while the NSW syllabus considers them to be Year 7 concepts.

Content

- 1 Rounding and estimating** 5NA099 U C
- 2 Multiplying numbers** 6NA123 U
- 3 Dividing numbers** 6NA123 U
- 4 Divisibility tests** 5NA098, 6NA122, 7NA149 U F PS R C

- Use simple divisibility tests
- Understand that if a number is divisible by a composite number then it is also divisible by the factors of that number

5 Long division NSW Stage 4 U

- Divide by a two-digit number

6 Extension: Roman numerals U C

- Recognise, read and convert Roman numerals used in everyday contexts, e.g. books, clocks, films

7 Powers and index notation 7NA149 U C

8 Square root and cube root 7NA150 U F R C

9 Prime and composite numbers 7NA149 U F R C

- Define and compare prime and composite numbers and explain the difference between them

10 Prime factors 7NA149 U

11 Highest common factor 7NA149 U F P S R C

- Solve problems involving lowest common multiples and greatest common divisors (highest common factors) for pairs of whole numbers, by comparing their prime factorisation

12 Lowest common multiple 7NA149 U F P S R C

13 Revision and mixed problems

Related topics

Year 7: Integers, Fractions and percentages, Algebra and equations, Decimals

Year 8: Pythagoras' theorem, Working with numbers, Algebra

Proficiency strands/Working mathematically

- **U = Understanding (knowing and relating maths):** Knowing specific concepts, notations and operations with whole numbers
- **F = Fluency (applying maths):** Applying appropriate strategies for different situations
- **PS = Problem solving (modelling and investigating with maths):** Solving problems using divisibility tests and prime factors (factor trees)
- **R = Reasoning (generalising and proving with maths):** Examining general principles in divisibility tests, prime numbers, greatest common divisors and lowest common multiples
- **C = Communicating (describing and representing maths):** Using index and root notation, and the language of factors and divisibility

Extension ideas

- Investigate larger Roman numerals and numeration systems of other cultures, including the use of zero and fractions.
- Investigate the history of calculation methods, for example Italian multiplication, and Newton's method for finding square roots.
- Learn the names of large numbers such as *trillion* (see the end of Exercise 3-07 on page 106) or introduce scientific notation.

- Examine how place value allows the algorithms to work (for example for addition, subtraction, long multiplication and long division).
- Investigate Goldbach's conjecture: every even number except 2 can be expressed as the sum of two primes.
- Research the history of Eratosthenes, Pascal and his triangle, Fibonacci and Fibonacci numbers in nature.
- Find divisibility tests for other numbers, such as 7 and 11 (see the end of Exercise 3-04 on page 100).
- How did mathematicians find square roots before calculators? Investigate Newton's method for calculating square roots, or examine irrational numbers and surds.

Teaching notes and ideas

- *Resources:* The library and the Internet, books on the history of numerals, calculators and spreadsheets
- Research the history of the Hindu-Arabic numerals and others, such as Greek, Mayan, Sumerian, Chinese, Papua New Guinean. How were fractions written in each of these number systems?
- Roman numerals are believed to be often used in television and film credits to make it more difficult to identify the date of production of the film as it becomes older and less current.
- Research the invention and history of zero.
- This is the first time students learn to divide by a two-digit number (long division). Reinforce estimation skills and the idea that division is the inverse of multiplication. See the NSW syllabus under 'Computation with Integers' for ideas.
- The GCD (HCF) is useful for simplifying fractions and factorising in algebra. The LCM is useful for adding and subtracting fractions.
- Research the history and achievements of Eratosthenes. The sieve works best when numbers are grouped in rows of six. See the Worksheet 'Sieve of Eratosthenes'.
- As an alternative to factor trees, prime factors can also be extracted by repeated division. See the Skillsheet 'Prime factors by repeated division'.
- Common mistake: $\sqrt{9} = \pm 3$. The square root of a number is a single *positive* value, so $\sqrt{9} = 3$ only. However, $-\sqrt{9} = -3$, and $\pm\sqrt{9} = \pm 3$.
- However, the cube root of a positive number is positive, but the cube root of a negative number is negative.
- The properties of the square and square root of ab will be covered in the Year 8 topic 'Working with numbers'.

Assessment ideas

- Research the history of number or one of the numeration systems.
- Vocabulary test
- An investigation project on one or more of the areas of this topic

Technology

Investigate powers and roots on scientific calculators, graphics calculators and spreadsheets. Use spreadsheets and CAS calculators to find greatest common divisors (GCD).

Language

- There is a lot of mathematical jargon in this topic, especially with words that have other everyday meanings, such as *prime*, *composite*, *factor* and *index*. How are the words *square* and *cube* in this topic related to their geometrical meanings?
- Examine the origin of mathematical symbols. The meaning and origin of the radical sign for square root and cube root may be explored.

4 Fractions and percentages

Time: 3 weeks (Term 2, Weeks 1–3)

Text: *New Century Maths 7*, Chapter 4, page 128

NSW/Australian Curriculum references: Number and Algebra

Fractions and Decimals 2/Fractions and decimals

- Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (6NA127)

Fractions, Decimals and Percentages/Real numbers

- Compare fractions using equivalence. Locate and represent fractions and mixed numbers on a number line. (7NA152)
- Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (7NA153)
- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (7NA154)
- Express one quantity as a fraction of another, with and without the use of digital technologies (7NA155)
- Connect fractions, decimals and percentages and carry out simple conversions (7NA157)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (7NA158)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-3WM** recognises and explains mathematical relationships using reasoning
- **MA4-5NA** operates with fractions, decimals and percentages

Introduction

This topic covers formal operations with fractions and some introductory work with percentages. In Years 5–6, students learned to: find equivalent fractions; simplify fractions; add and subtract fractions with related denominators (where one is a multiple of the other); convert between fractions, decimals and percentages; and find simple fractions and percentages of a quantity. Although the advancement of computers and the metric system has made decimals more practical than fractions, fraction skills are still applied in areas such as algebraic fractions, solving equations, ratios and similar figures.

Content

1 Fractions 7NA152 U F C

- Reduce a fraction to its simplest form (NSW Stage 4)
- Express improper fractions as mixed numerals and vice-versa (NSW Stage 4)

2 Ordering fractions 7NA152 U F C

3 Adding and subtracting fractions 7NA153 U F C

4 Adding and subtracting mixed numerals 7NA153 U F C

5 Fraction of a quantity 6NA127 U F C

- 6 **Multiplying fractions** 7NA154 U F R C
- 7 **Dividing fractions** 7NA154 U F R C
- 8 **Fraction problems** 7NA154 F P S C
- 9 **Fractions and the calculator** 7NA154 F P S
- 10 **Percentages, fractions and decimals** 7NA157 U F C
- 11 **Percentage of a quantity** 7NA158 U F C
- 12 **Expressing quantities as fractions and percentages** 7NA155, 7NA158 U F C
- 13 **Revision and mixed problems**

Related topics

Year 7: Integers, Whole numbers, Algebra and equations, Decimals, Analysing data, Probability, Ratios, rates and time

Year 8: Working with numbers, Algebra, Fractions and percentages, Investigating data, Equations, Probability, Ratios, rates and time

Proficiency strands/Working mathematically

- **U = Understanding (knowing and relating maths):** Learning the concepts, notations and operations of fractions and percentages
- **F = Fluency (applying maths):** Applying appropriate fraction and percentage skills to different situations
- **PS = Problem solving (modelling and investigating with maths):** Solving a variety of real-life problems using fractions
- **R = Reasoning (generalising and proving with maths):** Generalising the rules for multiplying and dividing fractions
- **C = Communicating (describing and representing maths):** Using the notations and terminology of fractions, decimals and percentages

Extension ideas

- History of fractions, fractions in ancient number systems, continued fractions
- Percentage increase and decrease, GST, discounts, profit and loss, interest, the unitary method (Year 8)

Teaching notes and ideas

- *Resources:* Cuisenaire rods; pattern blocks; newspaper cuttings of applications of percentages, for example interest rates, statistical yearbooks and opinion polls; calculators and spreadsheets.
- Some students may have trouble seeing why, for example, $\frac{1}{3} > \frac{1}{5}$, since $3 < 5$.
- Use the Worksheet 'Pop stick calculator' to make 'icy-pole stick' calculators for fractions.
- Learn the decimal and percentage equivalents of commonly-used fractions.
- Spend considerable time revising equivalent fractions and simplifying fractions, because success in this topic hinges upon mastery of these basic skills.

- Applications of fractions: adding fractions of an hour for payroll calculations, multiplying for overtime, fractions of ingredients in a cooking recipe, and converting recipes for different sized serves.
- Show that multiplication of fractions is easier if simplification occurs first.
- Why is division by a fraction equivalent to multiplication by its reciprocal?
- Students were introduced to the concept of a percentage in Years 5–6.
- Students can make a collage of newspaper clippings on the applications of percentages. Examine an advertising claim that uses percentages.

Assessment ideas

- Poster showing applications of fractions and percentages, or the history of fractions and percentages
- Writing activities involving fraction stories
- Open-ended questions, for example: ‘Find two fractions whose product is $\frac{4}{5}$.’

Technology

Use spreadsheets to convert fractions to decimals and percentages.

Language

- From the NSW syllabus, under ‘Fractions, Decimals and Percentages’: ‘The word fraction comes from the Latin word *frangere*, meaning “to break”.’
- Fractions of the form $\frac{a}{b}$ are actually called *common fractions*. Decimals, percentages and ratios are other types of fractions.
- The *reciprocal* of a number is the value which if multiplied by that number gives the product of 1. What is the everyday meaning of the word ‘reciprocal’?
- The word *cent* comes from the Latin *centum* meaning ‘one hundred’, so *per cent* means ‘out of one hundred’. The % symbol is a modified form of ‘ $\frac{\quad}{100}$ ’.

5 Algebra and equations

Time: 3 weeks (Term 2, Weeks 4–6)

Text: *New Century Maths 7*, Chapter 5, page 170

NSW/Australian Curriculum reference: Number and Algebra

Computation with Integers/Number and place value

- Apply the associative, commutative and distributive laws to aid mental and written computation (7NA151)

Algebraic Techniques 1 and 2/Patterns and algebra

- Introduce the concept of variables as a way of representing numbers using letters (7NA175)
- Create algebraic expressions and evaluate them by substituting a given value for each variable (7NA176)
- Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (7NA177)

Equations/Linear and non-linear relationships

- Solve simple linear equations (7NA179)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-3WM** recognises and explains mathematical relationships using reasoning
- **MA4-8NA** generalises number properties to operate with algebraic expressions
- **MA4-10NA** uses algebraic techniques to solve simple linear and quadratic equations

Introduction

Algebraic rules and procedures often appear meaningless to students if we do not take their level of mental development into account. The Australian Curriculum introduces algebra to Year 7 by generalising number laws and patterns, especially the commutative, associative and distributive laws of arithmetic. This topic covers elementary algebraic concepts, such as variables, algebraic abbreviations and substitution. Students are also introduced briefly to simple equations and three methods of solution: ‘guess, check and improve’, balancing, and backtracking. The next algebra topic, in Year 8, will cover more formal algebraic operations.

Content

- 1 The laws of arithmetic** 7NA151 U F R C
 - Understand that arithmetic laws provide powerful ways of describing and simplifying calculations and that using these laws leads to the generality of algebra
- 2 The distributive law** 7NA151 U F R C
- 3 Variables** 7NA175, 7NA177 U F R C
 - Recognise and use equivalent algebraic expressions (NSW Stage 4)
- 4 From words to algebraic expressions** 7NA177 U F P S R C
 - Move fluently between algebraic and word representations as descriptions of the same situation
- 5 Substitution** 7NA176 U F P S C
- 6 Equations** 7NA179 U F R C

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|---|--------------------|--------|-----------|
| 7 | One-step equations | 7NA179 | U F R |
| 8 | Two-step equations | 7NA179 | U F R |
| 9 | Equation problems | 7NA179 | F P S R C |

- Solve real-life problems by using pronumerals to represent unknowns

10 Revision and mixed problems

Related topics

Year 7: Integers, Whole numbers

Year 8: Algebra, Equations, Graphing linear equations

Proficiency strands/working mathematically

- **U = Understanding (knowing and relating maths):** Learning algebraic concepts and operations
- **F = Fluency (applying maths):** Applying algebra effectively to describe general rules and solve equations and problems
- **PS = Problem solving (modelling and investigating with maths):** Using equations to model and solve a problem
- **R = Reasoning (generalising and proving with maths):** Using algebra to describe general rules about operations with numbers, analysing the logic behind methods of solving equations
- **C = Communicating (describing and representing maths):** Understanding the concept of a variable and algebraic notation to generalise rules and equations

Extension ideas

- History of algebra
- The formal laws of arithmetic
- Adding and subtracting like terms, multiplying and dividing terms
- Graphical solutions of equations
- Substitution involving harder expressions and formulas
- Equations and formulas
- Harder equations, for example those with variables on both sides, with grouping symbols, with x^2 or $\frac{1}{2}$

Teaching notes and ideas

- *Resources:* counters, cubes, cups, blocks, envelopes, two-pan scales and other concrete materials for modelling variables in equations
- Algebra can be a difficult topic for students to understand. Spend time developing each skill before moving on; practise and revise often.
- Aim to teach this topic at an elementary level suitable for the ability of your class. Keep examples simple at Year 7 level. Avoid the temptation to jump into more formal and complex algebraic operations and equations, such as collecting like terms, expanding and factorising (Year 8), unless your students are advanced.
- Stress that a variable does not stand for an object but for the *number* of objects. Demonstrate the usefulness and power of variables in formulas, such as the formulas for the perimeter of a square and rectangle.

- Even if we do not know the value of a variable or term, we can still collect them. For example, ‘3 lots of x plus 4 lots of x equals 7 lots of x ’.
- Determine and justify whether a simplified or equivalent expression is correct by substituting a number.
- Stress that the goal of solving an equation is to have the variable on its own on the left-hand side of the equation and the value on the right-hand side.
- ~~The balancing and backtracking methods of solving equations are quite similar when written algebraically; the difference is in their models (and explanation).~~
- The process of ‘undoing’ (backtracking) or balancing needs to be explained and reinforced early. Use a ‘putting on socks and shoes’ analogy to explain why ‘undoing’ an equation must be performed in precise reverse order. We ‘undo’ the last thing first.
- When solving a word problem, identify the unknown quantity and give it a name, such as x . After solving, check that the solution sounds reasonable.

Assessment ideas

- Research assignment or poster on the laws of arithmetic, the history/meaning of algebra
- Writing activity on the use of variables or the method(s) of solving an equation

Technology

Spreadsheets, graphics calculators and GeoGebra can be used in substitution and ‘guess, check and improve’ solutions for equations. CAS (computer algebra systems) calculators can be used to simplify expressions, evaluate expressions and solve equations.

Language

- *Algebra* comes from the Arabic word ‘al-jabr’, meaning ‘restoration’ or the process of adding the same amount to both sides of an equation. In 825 CE, the Arabic mathematician al-Khwarizmi wrote a book called *Hisab al-jabr w'al-muqabala* (*The science of equations*).
- *Commutative* comes from ‘commute’, which means travelling between two places, and the commutative laws involve operating with two numbers in any order.
- *Associative* comes from ‘association’, which means a partnership or union, and the associative laws involve operating with three or more numbers in any order.
- *Distributive* comes from ‘distribute’, which means to spread out, and the distributive law involves spreading out a difficult multiplication across two simpler multiplications.
- *Variable* means ‘can be changed’; *pronumeral* means ‘stands for a numeral’; *substitute*: means ‘replace’ (as in basketball); *evaluate* means ‘to find the value of’; *simplify* means ‘write in shortest, reduced form’; *expression* refers to a ‘phrase’ involving terms and arithmetic operations, such as $2a + 5$; *equation* refers to a mathematical ‘sentence’ involving an expression and an equals sign, such as $2a + 5 = 13$.
- Encourage students to set out their equations neatly with equals signs aligned in the same column.

6 Geometry

Time: 3 weeks (Term 2, Weeks 7–9)

Text: *New Century Maths 7*, Chapter 6, page 206

NSW/Australian Curriculum references: Measurement and Geometry

Two-dimensional Space 2/Location and transformation

- Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies (6MG142)

Properties of Geometrical Figures 1/Geometric reasoning

- Classify triangles according to their side and angle properties and describe quadrilaterals (7MG165)
- Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (7MG166)

Properties of Geometrical Figures 1/Location and transformation

- Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries. (7MG181)

NSW Stage 4 outcomes

A student:

- MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- MA4-2WM** applies appropriate mathematical techniques to solve problems
- MA4-3WM** recognises and explains mathematical relationships using reasoning
- MA4-17MG** classifies, describes and uses the properties of triangles and quadrilaterals, and determines congruent triangles to find unknown lengths and angles

Introduction

This topic revises and extends transformation and symmetry concepts before introducing geometrical terminology and properties associated with triangles and quadrilaterals. While students have conducted some exploratory work with two-dimensional figures in primary school, in this topic they begin examining some more formal definitions and properties, including angle sums.

Content

- | | | |
|---|----------------|----------|
| 1 Transformations | 6MG142 | U C |
| 2 Composite transformations | 6MG142, 7MG181 | U F R C |
| <ul style="list-style-type: none"> Describe patterns and investigate different ways to produce the same transformational changes, such as using two successive reflections to provide the same result as a translation | | |
| 3 Line symmetry | 7MG181 | U F |
| 4 Rotational symmetry | 7MG181 | U F |
| 5 Classifying triangles | 7MG165 | U F R C |
| 6 Angle sum of a triangle | 7MG166 | U F PS R |
| 7 Exterior angle of a triangle | 7MG166 | U F PS R |
| 8 Classifying quadrilaterals | 7MG165 | U F R C |
| <ul style="list-style-type: none"> Describe squares, rectangles, rhombuses, parallelograms, kites and trapeziums | | |

- | | | |
|--|--------|----------|
| 9 Angle sum of a quadrilateral | 7MG166 | U F PS R |
| 10 Properties of quadrilaterals | 7MG165 | U F R C |
| 11 Revision and mixed problems | | |

Related topics

Year 7: Angles, The number plane

Year 8: Pythagoras' theorem, Geometry, Area and volume, Congruent figures

Proficiency strands/working mathematically

- **U = Understanding (knowing and relating maths):** Learning geometrical concepts, definitions, terminology and notation
- **F = Fluency (applying maths):** Applying correct procedures, language and notation to solve geometrical problems
- **PS = Problem solving (modelling and investigating with maths):** Find unknown angles in geometrical problems
- **R = Reasoning (generalising and proving with maths):** Using logic and reasoning to explore and deduce geometrical ideas and properties
- **C = Communicating (describing and representing maths):** Using the terminology of transformations, triangles, quadrilaterals and their properties

Extension ideas

- Investigate the history of geometry, Euclid
- Explore tangrams, polyominoes, tessellations, Escher art
- Convex versus non-convex quadrilaterals (Year 8)
- Angle sum of a polygon, inclusive properties of quadrilaterals (for example, a square is a special type of rectangle) (Year 9)
- From the NSW syllabus, under 'Properties of Geometrical Figures 1': 'Students who recognise class inclusivity and minimum requirements for definitions may address this Stage 4 outcome concurrently with Stage 5 Space and Geometry outcomes, where properties of triangles and quadrilaterals are deduced from formal definitions.' For example, is a rhombus a trapezium?
- Deductive geometry problems involving properties of triangles and quadrilaterals

Teaching notes and ideas

- *Resources:* ruler, set squares, compasses, protractors, paper and scissors, charts and posters
- From the NSW syllabus, under 'Properties of Geometrical Figures 1': 'At this Stage, the treatment of triangles and quadrilaterals is still informal, with students consolidating their understandings of different triangles and quadrilaterals and being able to identify them from their properties.' A range of examples of the various triangles and quadrilaterals should be given, including quadrilaterals containing a reflex angle and figures presented in different orientations.
- Proving properties of quadrilaterals by congruent triangles will occur in the Year 8 topic 'Congruent figures'.

- Give examples and counter-examples of the types of triangles and ask students to describe them in their own words. You may like to give the meaning first, then the name.
- Aim to make this topic as practical as possible. Students should be encouraged to cut out triangles and quadrilaterals and then measure and fold them to discover their properties.
- Draw triangles and quadrilaterals in different orientations: avoid horizontal bias.

Assessment ideas

- Vocabulary test
- ‘Which quadrilateral am I?’ puzzles
- Writing activities or a poster summary on the properties of triangles and/or quadrilaterals

Technology

There is much scope in this topic to use dynamic geometry software such as GeoGebra. The Internet is full of dynamic geometry animations that demonstrate the properties of triangles and quadrilaterals shown in this topic.

Language

- Use the correct terminology for transformations: a shape is reflected *across* or *in* a line (the line of reflection or axis of reflection), or rotated *about* a point (the centre of rotation).
- From the NSW syllabus, under Stage 3 ‘Two-dimensional Space 1’: ‘A shape is said to have rotational symmetry if a tracing of the shape matches it after the tracing is rotated part of a full turn.’
- *Equilateral* comes from the Latin *aequus latus*, meaning ‘equal side’, *isosceles* comes from the Greek *isos skelos*, meaning ‘equal legs,’ and *scalene* comes from the Greek *skalenos skelos*, meaning ‘uneven leg’.
- Avoid the term *base angles* in an isosceles triangle because it may be misleading. Instead, use *the angles opposite the equal sides* or *the two angles at the ends of the side that is not equal to the other sides*.

7 Decimals

Time: 3 weeks (Term 3, Weeks 1–3)

Text: *New Century Maths 7*, Chapter 7, page 268

NSW/Australian Curriculum references: Number and Algebra

Fractions and Decimals 1/Fractions and decimals

- Compare, order and represent decimals (5NA105)

Fractions and Decimals 2/Fractions and decimals

- Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers (6NA128)
- Multiply and divide decimals by powers of 10 (6NA130)

Fractions, Decimals and Percentages/Real numbers

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (7NA154)
- Round decimals to a specified number of decimal places (7NA156)
- Connect fractions, decimals and percentages and carry out simple conversions (7NA157)
- Investigate terminating and recurring decimals (8NA184)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-3WM** recognises and explains mathematical relationships using reasoning
- **MA4-5NA** operates with fractions, decimals and percentages

Introduction

Since the introduction of the metric system and computer technology, decimals have replaced the common fraction as the convention for representing parts of whole numbers. The algorithms from primary school for operating with whole

numbers are now extended to decimals. The first part of this topic revises decimal skills from primary school so, for a more advanced Year 7 class, this may be revised briefly and/or using a class assignment. Estimation skills and mental computation strategies should be promoted and reinforced regularly.

Content

- 1 Ordering decimals** 5NA105 U F C
- 2 Decimals and fractions** 7NA157 U F C
 - Convert terminating decimals to fractions
- 3 Adding and subtracting decimals** 6NA128 U F PS C
- 4 Multiplying and dividing decimals by powers of 10** 6NA130 U F C
- 5 Multiplying decimals by estimating** 7NA154 U F
- 6 Multiplying decimals** 7NA154 U F C
- 7 Dividing decimals by whole numbers** 7NA154 U F PS R

8 Dividing decimals 7NA154 U F P S R C

9 Terminating and recurring decimals 7NA157, 8NA184 U F R C

10 Rounding decimals 7NA156 U F P S C

11 Decimal problems 7NA154 F P S C

12 Revision and mixed problems

Related topics

Year 7: Whole numbers, Fractions and percentages, Area and volume, Analysing data, Probability, Ratios, rates and time

Year 8: Pythagoras' theorem, Working with numbers, Fractions and percentages, Investigating data, Probability, Ratios, rates and time

Proficiency strands/Working mathematically

- **U = Understanding (knowing and relating maths):** Understanding decimal concepts, operations and terminology
- **F = Fluency (applying maths):** Applying appropriate techniques and operations with decimals for different situations
- **PS = Problem solving (modelling and investigating with maths):** Solving real-life problems using decimal operations
- **R = Reasoning (generalising and proving with maths):** Understanding the logic behind decimal multiplication and division; observing general patterns in terminating and recurring decimals
- **C = Communicating (describing and representing maths):** Using the language and notation of terminating and recurring decimals

Extension ideas

- Investigate patterns in the recurring decimals of fraction families
- Investigate the value of $0.\dot{9}$. Is it really equal to 1?
- How are decimals written in other number systems? How are decimals used in scientific notation?
- Converting recurring decimals to fractions (Year 9, Stage 5.3)

Teaching notes and ideas

- In primary school, students learn to write, order, add and subtract decimals, multiply and divide decimals by any number from 2 to 9 and powers of 10 (terminating decimal answers only), and multiply decimals by two-digit numbers.
- Investigate the Dewey classification system for classifying books, as an application of ordering decimals. See the Worksheet 'Dewey decimals'.
- Decimals can be ordered by comparing digits in the same decimal place. Ask students to sort athletic distances/times and place them in order.
- For operations with decimals, encourage students to estimate answers first.
- Applications of decimals: shopping, buying fruit, meat, petrol; calculating wages; car odometers (tenths of a kilometre); stopwatches.

- Some decimals are *neither* terminating nor recurring. Their digits run endlessly, but without repeating. For example, $\sqrt{2} = 1.4142135 \dots$ and $\pi = 3.1415926 \dots$
- Rounding may be introduced by examining examples of rounding to the nearest dollar, the nearest cent, the nearest 5 cents, the nearest centimetre or the nearest whole number. One-cent and two-cent coins were phased out in Australia in 1990.
- When teaching rounding, include more difficult examples, such as rounding 4.8971 to two decimal places.

Assessment ideas

- Topic assignment

Technology

Students may investigate the FIX mode on a calculator or the 'Format cell' function on a spreadsheet for rounding decimals. Students can use the spreadsheet to order decimals and can explore converting fractions to terminating and recurring decimals.

Language

- *deci* means tenth (for example, *decimate*, *decimetre*, *decibel*). The word *decimal* when used to describe fractions is actually short for 'decimal fraction'.
- An anagram of 'A DECIMAL POINT' is 'I'M A DOT IN PLACE'.
- From the NSW syllabus, under Stage 3 'Fractions and Decimals 2': 'The decimal 1.12 is read "one point one two" and not "one point twelve".'
- *Terminating* means 'stopping'; *recurring* means 'repeating'.
- Note the different ways rounding is described: 'Approximate', 'Write correct to', 'to two decimal places', 'to the nearest hundredth'.
- As with large numbers, when writing long decimals leave a space after every three digits (for example, 3.141 592 65).

8 Area and volume

Time: 3 weeks (Term 3, Weeks 4–6)

Text: *New Century Maths 7*, Chapter 8, page 302

NSW/Australian Curriculum references: Number and Algebra

Area 1/Using units of measurement

- Calculate the perimeter and area of rectangles using familiar metric units (5MG109)

Length 2/Using units of measurement

- Connect decimal representations to the metric system (6MG135)
- Convert between common metric units of length, mass and capacity (6MG136)

Volume and Capacity 2/Using units of measurement

- Connect volume and capacity and their units of measurement (6MG138)

Area/Using units of measurement

- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (7MG159)

Volume/Using units of measurement

- Calculate volumes of rectangular prisms (7MG160)
- Choose appropriate units of measurement for area and volume and convert from one unit to another (8MG195)

Volume/Shape

- Draw different views of prisms and solids formed from combinations of prisms (7MG161)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-12MG** calculates the perimeters of plane shapes and the circumferences of circles
- **MA4-13MG** uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area
- **MA4-14MG** uses formulas to calculate the volumes of prisms and cylinders, and converts between units of volume

Introduction

In this Measurement topic, perimeter, area and volume concepts are revised and extended. The emphasis is upon understanding the meaning of each concept and the decimal relationships between their units. This topic could be partly assessed by a practical test. Note that this topic also ties Measurement with Geometry (prisms) and Algebra (formulas).

Content

1 The metric system 6MG135, 6MG136 U F C

- Recognise the significance of the metric prefixes in units of measurement
- Identify and use the correct operations when converting units, including millimetres, centimetres, metres, kilometres, milligrams, grams, kilograms, tones, millilitres, kilolitres and megalitres

2 Perimeter 5MG109 U F PS R

- 3 **Metric units for area** 8MG195 U F PS R C
- 4 **Area of a rectangle** 7MG159 U PS R
- 5 **Area of a triangle** 7MG159 U PS R
- 6 **Area of a parallelogram** 7MG159 U PS R
- 7 **Areas of composite shapes** 7MG159 U F PS C
- 8 **Drawing prisms** 7MG161 U F C
- 9 **Metric units for volume** 8MG195 U F PS R C
- 10 **Volume of a rectangular prism** 7MG160 U F PS R
- 11 **Volume and capacity** 6MG136, 6MG138 U F PS C
 - Recognise that 1 mL is equivalent to 1 cm³
- 12 **Revision and mixed problems**

Related topics

Year 7: Algebra and equations, Geometry, Decimals, Ratios, rates and time

Year 8: Pythagoras' theorem, Working with numbers, Algebra, Geometry, Area and volume, Ratios, rates and time

Proficiency strands/Working mathematically

- **U = Understanding (knowing and relating maths):** Learning measurement concepts, terminology and techniques
- **F = Fluency (applying maths):** Selecting correct strategies to convert between metric units and calculate areas and volumes
- **PS = Problem solving (modelling and investigating with maths):** Solving problems involving measurement, perimeter, area and volume
- **R = Reasoning (generalising and proving with maths):** Introducing formulas to generalise the rule for calculating perimeters, areas and volumes; analyse relationships when converting between metric units for length, area and volume
- **C = Communicating (describing and representing maths):** Knowing and applying the names of the metric units

Extension ideas

- Research the history of measurement, the imperial and metric systems
- Pi and the circumference of a circle, areas of special quadrilaterals and circles, volumes of right prisms (8MG196, 8MG197, 8MG198)
- Surface area of prisms
- Investigate the relationships between volume, capacity and mass of water. Demonstrate that one litre of water has a mass of 1 kg.

Teaching notes and ideas

- *Resources:* rulers, trundle wheels, 1 cm grid paper and transparencies, centicubes, measuring containers, base 10 blocks, models and nets of prisms, different-shaped boxes, square dot paper, isometric dot paper
- Demonstrate the usefulness and power of variables in formulas, such as the formulas for the perimeters of a square and rectangle.

- Areas may be found by paper-cutting activities and grid overlays: print out the Worksheet '1 cm grid paper' and photocopy it onto an overhead transparency.
- Measure the perimeters and areas of various places around the school: playgrounds, basketball courts, football fields, or library spaces. Estimate areas of windows, noticeboards, blackboards, desktops, postage stamps. Mark a square metre or a hectare on the school grounds.
- Applications of area: bricks, tiling, wallpaper, or carpeting. Investigate brick walls to determine the number of bricks per square metre. Investigate areas of rooms, homes, or blocks of land.
- Although students have calculated perimeters, areas and volumes of shapes in primary school, this is the first time they deal with the algebraic formulas for doing so. This is the first application of algebra concepts (variables) from the 'Algebra and equations' topic.
- Draw triangles in different orientations, and include obtuse triangles
- Investigate alternative ways of finding composite areas: L-shape, T-shape, U-shape, and trapezium
- Many students have trouble drawing solid shapes, so guidance should be given. At this age, many Year 7 students have difficulty visualising and drawing objects in 3D, especially from different perspectives
- Compare measured volumes with labelled capacities of juice packs
- Students should not be fooled into thinking that tall, thin containers hold 'more' liquid than short, wide containers

Assessment ideas

- A practical test for measuring perimeter, area and volume
- A practical project involving outdoor investigation or problem-solving

Technology

Use a spreadsheet to find the largest possible rectangle (in area) for a given perimeter (see 'Technology', page 323 in the textbook). Use the Internet to investigate land areas of countries or Australian states.

Language

- Discuss the meanings of the prefixes *milli*, *centi*, *kilo*, etc.
- Note the different types of length: height, width, breadth, depth, distance, thickness.
- In this chapter, we have used the word and abbreviation *width* (*w*) rather than *breadth* (*b*) for two reasons. 'Width' is more commonly-understood than 'breadth', and the variable *b* is already used for 'base'.
- The metric unit of length is spelt *metre*, not *meter*. 'Meter' is the US spelling, or refers to a measuring device.
- From Stage 4 of the NSW syllabus under 'Area', and 'Volume', respectively: 'The abbreviation m^2 is read "square metre(s)" and not "metre squared" or "metre square" ... The abbreviation m^3 is read "cubic metre(s)" and not "metre cubed" or "metre cube".'
- Trivia: The metric unit of area called *are* equals $100 m^2$ so one hectare equals 100 ares (*hecto* means 100).

9 The number plane

Time: 3 weeks (Term 3, Weeks 7–9)

Text: *New Century Maths 7*, Chapter 9, page 364

NSW/Australian Curriculum references: Number and Algebra, Measurement and Geometry

Position 1/Location and transformation

- Use a grid reference system to describe locations (5MG113)

Position 2/Location and transformation

- Introduce the Cartesian coordinate system using all four quadrants (6MG143)

Linear Relationships/Linear and non-linear relationships

- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (7NA178)

Linear Relationships/Location and transformation

- Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates (7MG181)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-3WM** recognises and explains mathematical relationships using reasoning
- **MA4-11NA** creates and displays number patterns; graphs and analyses linear relationships; and performs transformations on the Cartesian plane

Introduction

This topic revises and extends number plane and transformation concepts learned in primary school. Number plane activities may be limited to locating and plotting points, rather than formal coordinate geometry. Locating position and plotting coordinates were introduced in Years 3–4 while the four-quadrant number plane (including negative coordinates) was introduced in Years 5–6. This Year 7 topic focuses on patterns found when plotting points on the

number plane and when shapes are translated, reflected and rotated on the number plane. The Year 8 topic ‘Graphing linear equations’ will examine graphs of algebraic rules on the number plane.

Content

1 Location using coordinates 5MG113 U F C

- find a place on a map or in a directory, given its coordinates (NSW)

2 The number plane 6MG143, 7NA178 U F R C

3 The number plane with negative numbers 6MG143, 7NA178 U F R C

4 Graphing tables of values 7NA178 U F R

- Plot points from an integer table of values and recognise simple patterns, such as points that lie on a straight line

5 Transformations on the number plane 7MG181 U F R C

- Plot and name the coordinates for P' resulting from translating P one or more times, or reflecting P in either the x - or y -axis, or rotating P by a multiple of 90° about the origin (NSW Stage 4)

6 Revision and mixed problems

Related topics

Year 7: Integers, Geometry

Year 8: Graphing linear equations

Proficiency strands/Working mathematically

- **U = Understanding (knowing and relating maths):** Understanding the concept and terminology of grid references and the number plane
- **F = Fluency (applying maths):** Plotting and reading coordinates of locations on the number plane
- **R = Reasoning (generalising and proving with maths):** Analysing general patterns found when locating and plotting points on the number plane and when transforming shapes on the number plane
- **C = Communicating (describing and representing maths):** Describing locations and transformations on the number plane using coordinates

Extension ideas

- From the NSW syllabus under 'Linear relationships': 'Descartes and Fermat used coordinates to identify points in terms of positive or zero distances from axes. Isaac Newton introduced negative values.'
- Investigate latitude and longitude on the world globe, or polar coordinates on a number plane.
- Graph points, lines and curves on the number plane (Years 8–9).

Teaching notes and ideas

- *Resources:* number line, number plane or grid paper, street map and grids, Battleship games, number plane picture puzzles
- Convert the classroom into a coordinate system of rows and columns.
- Stress that order is important with coordinates, for example (2, 5) is not (5, 2). Hence the term *ordered pair*.
- Note that the coordinates on a number plane describe a *point*, not a rectangular cell as in a road map or a Battleship game.

Assessment ideas

- Research assignment or poster on the number plane
- Coordinates and/or terminology test

Technology

Play Battleship games on the computer. Investigate GPS and global coordinates (latitude, longitude).

Language

- From the NSW syllabus under Stage 3 'Position 2': 'The Cartesian plane (commonly referred to as "the number plane") is named after [René] Descartes who was one of the first to develop analytical [coordinate] geometry on the number plane.'
- Examine the everyday meanings of the words *origin*, *quadrant*, *coordinate*, and how they relate to their mathematical meanings.

10 Analysing data

Time: 3 weeks (Term 4, Weeks 1–3)

Text: *New Century Maths 7*, Chapter 10, page 394

NSW/Australian Curriculum references: Statistics and Probability

Data 2/Data representation and interpretation

- Interpret secondary data presented in digital media and elsewhere (6SP148)

Single Variable Data Analysis 1/Data representation and interpretation

- Identify and investigate issues involving numerical data collected from primary and secondary sources (7SP169)
- Construct and compare a range of data displays including stem-and-leaf plots and dot plots (7SP170)

Single Variable Data Analysis 2/Data representation and interpretation

- Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of the data. (7SP171)
- Describe and interpret data displays using median, mean and range (7SP172)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-3WM** recognises and explains mathematical relationships using reasoning
- **MA4-19SP** collects, represents and interprets single sets of data, using appropriate statistical displays
- **MA4-20SP** analyses single sets of data using measures of location and range

Introduction

This Statistics topic revises different types of graphs from primary school before introducing dot plots, stem-and-leaf plots, statistical measures and analysing data. This is a practical topic, and it is expected that some data will be generated from student surveys undertaken in class. The mass media, including the Internet, is a rich source of data for statistical investigation.

Content

1 Interpreting graphs 7SP170 U F PS R C

- Understand that some data representations are more appropriate than others for particular data sets, and answer questions about those data sets

2 Misleading graphs 6SP148 U F PS R C

- Identify potentially misleading data representations in the media, such as graphs with ‘broken’ axes or non-linear scales, or graphics not drawn to scale

3 Dot plots 7SP170 U F R C

4 Stem-and-leaf plots 7SP170 U F R C

5 The mean and mode 7SP171 U F PS R C

6 The median and range 7SP171 U F PS R C

- Use the mean and median to compare data sets and explain how outliers may affect the comparison

7 Analysing dot plots and stem-and-leaf plots 7SP169, 7SP170 U F PS R C

8 Comparing data sets 7SP169, 7SP172 U F PS R C

9 Revision and mixed problems

Related topics

Year 7: Fractions and percentages, Probability, Ratios, rates and time (travel graphs)

Year 8: Investigating data, Probability

Proficiency strands/Working mathematically

- **U = Understanding (knowing and relating maths):** Knowing the various types of data displays and statistical measures
- **F = Fluency (applying maths):** Reading and interpreting graphs, calculating and analysing statistics, comparing data sets
- **PS = Problem solving (modelling and investigating with maths):** Analysing data to solve problems, drawing conclusions
- **R = Reasoning (generalising and proving with maths):** Making generalisations and drawing conclusions from statistical displays and measures
- **C = Communicating (describing and representing maths):** Representing data sets on different types of statistical graphs and displays

Extension ideas

- Frequency tables, histograms, polygons (Year 8)
- The mean from a frequency table, the mean using the calculator's statistics mode (Year 8)
- Quartiles, interquartile range (Year 10)

Teaching notes and ideas

- *Resources:* Spreadsheets, statistical and graphing software, statistical yearbooks and census data from the Australian Bureau of Statistics, graphs and data from newspapers and magazines, RTA accident statistics.
- Compare the strengths and weaknesses of different types of graphs for use with the same set of data.
- Collect examples of misleading graphs and the misuse of statistics in the media, including from the Internet.
- Sampling, data collection, types of data, frequency histograms and polygons. The statistics mode of a calculator will be met in the Year 8 topic 'Investigating data'.
- The class may be surveyed on a number of characteristics: height, arm span, shoe size, heartbeat rate, reaction time, number of children in family, number of people living at home, hours slept last night, number of letters in first name, number of cars or mobile phones owned at home, make/colour of car, mode of travel to school, favourite television/radio station, reaction time, eye/hair colour, birth month or star sign.
- In Australia, a census takes place every five years, in a year ending in 1 or 6.
- Students learned to calculate the average of a list of numbers from primary school, but they may not know the word *mean*.

- Applications of mean: sports averages, rainfall or temperatures, number of matches in a matchbox, market research.
- Applications of mode: number of people in an Australian family, most popular Australian car, ordering stock for a shop.
- Applications of median: wages, house prices.
- The range is a measure of spread. Application: daily temperature range.
- When is it more appropriate to use the mode or median, rather than the mean, when analysing data? Which is higher, the mean or median price of Australian houses?

Assessment ideas

- Include open-ended questions: The range of a set of eight scores is 10 and the mode is 3. What might the scores be?
- Plan, implement and report on a statistical investigation.
- Vocabulary test
- Investigate the use and abuse of statistics and statistical graphs in the media.

Technology

Explore the statistical and graphing features of a spreadsheet, GeoGebra, Fx-Stat, a graphics/CAS calculator or software. Visit the Australian Bureau of Statistics website (www.abs.gov.au) particularly the CensusAtSchool page (www.abs.gov.au/censusatschool) website, or purchase the CD-ROMs.

Language

- What is the difference between a *graph* and a *plot*?
- *Median* means middle, for example median strip on a highway, or sounds like 'medium'. *Mode* (French) means fashionable, or popular.
- The mean, median and mode are collectively called *measures of location* or *measures of central tendency*.
- This topic contains a lot of statistical jargon, and a student-created glossary may be useful.

11 Probability

Time: 2 weeks (Term 4, Weeks 4–6)

Text: *New Century Maths 7*, Chapter 11, page 438

NSW/Australian Curriculum references: Statistics and Probability

Chance 1/Chance

- Recognise that probabilities range from 0 to 1 (5SP117)

Chance 2/Chance

- Conduct chance experiments with both small and large numbers of trials using appropriate digital technologies (6SP145)
- Compare observed frequencies across experiments with expected frequencies (6SP146)

Probability 1/Chance

- Construct sample spaces for single-step experiments with equally likely outcomes (7SP167)
- Assign probabilities to the outcomes of events and determine probabilities for events (7SP168)
- Identify complementary events and use the sum of probabilities to solve problems (8SP204)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-21SP** represents probabilities of simple and compound events

Introduction

This short topic revises and extends probability concepts learned in primary school, such as the language of chance, sorting events in order of likelihood and listing the possible outcomes of a chance situation. Students will develop a formal understanding of probability as a calculated value, and compare this against experimental probability (relative frequency). There are many opportunities here for class discussion, practical lessons and language activities. This topic will also allow students to apply their number skills with fractions, decimals and percentages.

Content

- 1 **Sample spaces** 7SP167 U C
- 2 **Probability** 7SP168 U F PS C
- 3 **The range of probability** 5SP117 U F PS C
- 4 **Experimental probability** 6SP145, 6SP146 U F PS R
- 5 **Complementary events** 8SP204 U F PS R C
- 6 **Revision and mixed problems**

Related topics

Year 7: Whole numbers, Fractions and percentages, Decimals, Analysing data

Year 8: Investigating data, Probability

Proficiency strands/Working mathematically

- **U = Understanding (knowing and relating maths):** Knowing the terminology, concepts and notations of probability
- **F = Fluency (applying maths):** Applying probability theory to solve problems
- **PS = Problem solving (modelling and investigating with maths):** Using probability theory to investigate problems, analyse the results of a chance experiment
- **R = Reasoning (generalising and proving with maths):** Making generalisations and inferences about probability situations and experiments, including complementary events
- **C = Communicating (describing and representing maths):** Describing sample spaces, probabilities and complementary events using the language of chance

Extension ideas

- Venn diagrams and two-way tables (Year 8)
- Two-stage or three-stage experiments: making lists, tables, tree diagrams
- Counting techniques
- Investigate probability expressed as odds (ratio)
- The addition rule of probability

Teaching notes and ideas

- *Resources:* dice, coins, counters, spinners, playing cards, probability simulation software
- Order cards showing chance words such as *unlikely* and *even chance* on an empty number line (see the Worksheet 'Chance cards'). Match situations or calculated probabilities to chance words. Ask students to write or tell a story, or complete a cloze passage, involving chance.
- Do not assume that all students have had experience with the properties of playing cards: suits, colours, deck of 52. Be sensitive to religious and cultural differences in attitudes towards gambling.
- Explain that a probability of $\frac{1}{4}$ means 'one chance in 4'.
- Reinforce the ideas of randomness and equally likely outcomes. Discuss the claim: 'Since traffic lights can show red, amber or green, the probability that a light shows red is $\frac{1}{3}$ '.
- Investigate common misconceptions about chance, such as if a coin is tossed and heads comes up five times in a row then, for the next toss, tails has a better chance than heads.
- Do not fall into the trap of thinking of (or teaching) probability as being all about games of chance and gambling. Investigate the applications of probability in insurance, for example car accidents, home burglaries, life expectancy, or quality control and sampling. Use the Internet to find quotes on premiums. What factors affect the chances of a particular car being stolen?
- Collect newspaper or Internet articles involving chance, and compare probabilities expressed as fractions, decimals and percentages. Test a chance game to see if it is fair.
- Investigate the frequency of each letter of the alphabet in print or in the Scrabble game.
- Investigate games involving dice (Craps, Yahtzee), coins (Two-Up), cards, raffles, spinners, Roulette. Play calculator cricket or noughts-and-crosses on the computer/Internet. Use real or simulated experiments to find the probabilities of winning and compare these with theoretical probabilities. Investigate the data from past Lotto draws using the NSW Lotteries website (www.nswlotteries.com.au).

Assessment ideas

- Vocabulary test or writing activities using the language of chance
- Research/investigation involving listing and counting the outcomes of a sample space
- Open-ended questions: Write an event that could have a probability of 65%
- Design a game or spinner that satisfies some probability specifications

Technology

Random numbers can be generated on a calculator, graphics or CAS calculator, or spreadsheet. The Internet, spreadsheets and other software may be used to simulate a chance situation such as a lotto draw, coin tosses and dice throws.

Language

- Interpret a probability story. What does '20% chance of rain on the weekend' really mean?
- Explain the meaning of the following terms: *equally likely*, *random*, *outcome*, *event*, *sample space*. What is the difference between *impossible* and *improbable*?
- Students should know the difference between an *outcome* and an *event*. An event contains one or more outcomes.
- How is the use of the word *complementary* in this topic similar to its use with *complementary angles* or its everyday English meaning? Carry out language activities that involve identifying the complement of an event such as 'There are fewer than 3 children in a family'. This could be done as a 'matching pairs' memory card game.
- What is the difference between 'more than 3' and '3 or more'?
- *Buckley's chance* is an old Australian expression meaning little or no chance. (See 'Just for the record' on page 452.)

12 Ratios, rates and time

Time: 3 weeks (Term 4, Weeks 6–9)

Text: *New Century Maths 7*, Chapter 12, page 464

NSW/Australian Curriculum references: Number and Algebra, Measurement and Geometry

Time 1/Using units of measurement

- Compare 12- and 24-hour time systems and convert between them (5MG110)

Time 2/Using units of measurement

- Interpret and use timetables (6MG139)

Proportion/Real numbers

- Recognise and solve problems involving simple ratios (7NA173)

Financial Mathematics/Money and financial mathematics

- Investigate and calculate ‘best buys’, with and without digital technologies (7NA174)

Proportion/Linear and non-linear relationships

- Investigate, interpret and analyse graphs from authentic data (7NA180)

Time/Using units of measurement

- Solve problems involving duration, including 12- and 24-hour time within a single time zone (8MG199)

NSW Stage 4 outcomes

A student:

- **MA4-1WM** communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols
- **MA4-2WM** applies appropriate mathematical techniques to solve problems
- **MA4-3WM** recognises and explains mathematical relationships using reasoning
- **MA4-6NA** solves financial problems involving purchasing goods
- **MA4-15MG** performs calculations of time that involve mixed units, and interpret time zones

Introduction

In this topic, students are introduced briefly to ratios, rates and time calculations. Ratios compare parts or shares of something, and are similar (but not identical) to fractions, so students will be able to apply their mental and calculator skills with fractions here. Rates compare quantities expressed in different units, for example speed compares the distance travelled with the time taken. Travel graphs and time calculations are included in this topic because travel graphs also compare distance with time, while many rates include units of time. Note that this topic links together concepts in Number, Measurement and Statistics (graphs, timetables).

Content

- 1 **Ratios** 7NA173 U C
- 2 **Simplifying ratios** 7NA173 U F C
- 3 **Ratio problems** 7NA173 U F P S C

4 Rates 7NA173 U C

5 Best buys 7NA174 U F PS R

6 Rate problems 7NA173 U F PS C

7 Travel graphs 7NA180 U F PS R C

- Interpret features of travel graphs, such as the slopes of lines and the meaning of horizontal lines

8 Time calculations NSW Stage 4 U F C

- Round calculator answers to the nearest minute or hour (NSW Stage 4)
- Add and subtract time mentally and with a calculator using the 'degrees, minutes, seconds' button (NSW Stage 4)

9 24-hour time 5MG110 U F C

10 Time differences 8MG199 U F PS

11 Timetables 6MG139 U F PS

12 Revision and mixed problems

Related topics

Year 7: Fractions and percentages, Decimals, Area and volume, Analysing data

Year 8: Fractions and percentages, Ratios, rates and time

Proficiency strands/working mathematically

- **U = Understanding (knowing and relating maths):** Learning the concepts and operations involving ratios, rates and time
- **F = Fluency (applying maths):** Applying appropriate concepts and skills to situations
- **PS = Problem solving (modelling and investigating with maths):** Solving real-life problems using ratios, rates, travel graphs and time calculations
- **R = Reasoning (generalising and proving with maths):** Making generalisations and inferences about best buys and travel graphs
- **C = Communicating (describing and representing maths):** Using the correct notations for ratios, rates, hours and minutes, and 24-hour time

Extension ideas

- Investigate the golden ratio and the golden rectangle: see the NSW syllabus under 'Proportion' (Stage 4)
- Dividing a quantity in a given ratio, scale drawings, international time zones (Year 8)
- Solve harder rate problems, such as fuel consumption, and converting rates to different units, for example from km/h to m/s
- Extend the unitary method to fraction and percentage problems
- Research the history of the calendar and/or time measurement: Julian, Gregorian, Islamic, Chinese, Jewish calendars, daylight saving, international time zones, International Date Line
- Research ancient time-measuring devices such as the hourglass, sundial, water clock, pendulum, candle clock

Teaching notes and ideas

- *Resources:* supermarket catalogues for best buys, tables of data showing rates such as fuel consumption or birth rates, stopwatch, 24-hour clock, calendars, timetables, map with world time zones
- Encourage the class to list instances of ratios, when the parts or shares of a mixture are important: cordial, punch, cake mix, lawn mower fuel, concrete, paste (flour and water), lemonade, milkshake, fertiliser, gear ratios, slopes of hills, probability and betting odds.
- Investigate scale diagrams. Scale maps, dividing a quantity in a given ratio, speed and international time zones will be covered in the Year 8 topic 'Ratios, rates and time'.
- Investigate the aspect ratios of television, computer and cinema screens (see 'Just for the record' on page 468).
- For rates, stress that the slash (/) indicates the division process and means 'per' or 'out of'.
- Encourage students to list examples of rates and the two units those rates compare: birth rate; population growth; heartbeat; typing speed; fuel consumption; postage rates; metric conversions and currency conversions; download speed; filling a tank; mobile phone costs; classified advertisements; cost of petrol, meat or fruit; population density; cricket run rate (runs/over); batter's strike rate (runs/100 balls); bowler's strike rate (balls/wicket); and other sports statistics.
- Investigate population density, population growth, birth rate, death rate, speed, and fuel consumption.
- Investigate unit pricing on supermarket shelves, and how sometimes the unit is 100 mL rather than 1 mL (Why?). Discuss why the 'best buy' is usually the largest item. Since 2009, unit pricing has been compulsory in all Australian supermarkets, so supermarket retailers must display prices of products according to their cost per unit whenever this is possible.
- Compare the advantages of 12-hour and 24-hour time and investigate their history. What do 'a.m.' and 'p.m.' stand for? When and where is 24-hour time used? Why is it also called 'military time'?
- Applications of time calculations: bus/plane trip using timetables, length of a movie, payroll (hours worked), sunrise to sunset, length of a school day or work day.
- Plan a holiday and create a travel schedule with the times written in 12-hour or 24-hour time

Assessment ideas

- Problem-solving test
- Poster assignment about applications of ratios or rates
- Travel graph 'tell me a story' writing activities
- Problems involving travel times and time zones

Technology

Ratios can be entered into a calculator using the $[a^b/c]$ fraction key. However, when simplifying 'improper ratios,' use the $[d/c]$ key to convert the mixed numeral answer to a 'proper ratio'. Students should be introduced to the calculator's 'degrees-minutes-seconds' key for time calculations. Use the Internet to find airline, train and cinema timetables. Put itineraries onto a spreadsheet and calculate different times.

Language

- A symbol for minute is ' ; a symbol for second is " . Their abbreviations are 'min' and 's' respectively.
- The word *minute* comes from the Latin *pars minuta prima*, meaning the first ('prima') division ('minuta') of an hour. In this way, it is related to the alternative meaning and pronunciation of the word minute as 'tiny'. The word *second* comes from *pars minuta secunda*, meaning the second ('secunda') division of an hour.
- The parts of a ratio are called its *terms*.
- Why does the *unitary method* have that name?