REGULAR AND REVERSE CLOSED CRUSHING CIRCUITS – A NEW MATHEMATICAL AND
GRAPHICAL PROCEDURE FOR THEIR ANALYSIS AND CONTROL

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ABSTRACT

The present work deals with new developments in calculating the performance of closed crushing circuits (regular and reverse). In previous works, some simple equations describing these circuits were derived. Here, these equations are interrelated to newly derived ones. The relationships are suitable for the construction of nomographs. The use of these nomographs facilitates calculations of circulating load and screen efficiency (undersize recovery). They can be used for the selection of crushing and sizing equipment. The procedure gives accurate and simple prediction of the circulating load without knowing any of the initially proposed equations. Test cases illustrate the above findings.

Keywords
Closed crushing circuits; regular and reverse closed circuits, comminution; screening; modelling, nomographs, graphical analysis and control

INTRODUCTION

Closed crushing circuits are arrangements controlling the maximum product size of the crushing process. There are two simple types of such circuits. The regular or direct circuit (new feed to the crusher) is usually fed with material of low undersize content, and the reverse or indirect (new feed to the screen) in the case where the feed contains a relatively high undersize percentage, which should be removed prior to its introduction into the crusher.

There have been many studies of closed crushing circuits. In these works, equations were established giving the circulating load \( R_g \) of the regular closed circuit, as function of screen efficiency \( E \) and the percent undersize content \( u \) in the crusher discharge (Karra, 1979; Stamboltzis, 1985). Another equation (Allis-Chalmers, 1953; Flavel, 1977) expressed \( R_g \) as function of the efficiency of removal \( e \) (percent oversize content in the retained on the screen) and the percent oversize content \( x = 100 - u \) in the crusher discharge. The circulating load \( R_g \) was also calculated (Testut, 1958; Magerowski and Karra, 1982; Stamboltzis, 1985) from the percent undersize contents \( u \), \( r \) in the crusher discharge and the retained on the screen, respectively.

In the reverse crushing circuit the equations giving the circulating load \( R_v \) are functions:

- Of \( E, t, u \) (Karra, 1979), where \( t, u \) are the undersize contents in the new feed and the crusher discharge, or of \( t, u \) and \( r \) (Stamboltzis, 1985), where \( t, u \) are as defined above and \( r \) is the undersize content in the retained on the screen.

In the present work it will be shown that the circulating loads \( R_g \) and \( R_v \) can be calculated from \( E \) and \( r \), by the same equation. The screen efficiency \( E \), in the regular circuit, will be shown to be a function of \( u, r \), while in the reverse closed circuit a function of \( t, u \) and \( r \).

The equations already proposed and those here derived will be properly recast and, from these expressions, nomographs will be constructed. Thus, the calculation of the various closed-circuit parameters becomes
simple and accurate. Hence, without knowing any equations, all the circuit parameters can be calculated. Also, the earlier proposed equation giving the circulating load ratio $R_g$ as a function of $e$ and $r$, is here extensively examined for its mathematical consistency.

Mathematical modelling

In what follows, we present the mathematical equations describing the two types of closed circuits.

a. Regular closed circuit

In the regular closed circuit (Fig. 1):

- $R_g$ is the circulating load ratio expressed as fraction of the new feed $F$
- $W_g$ is the circulating load tonnage in tons/h
- $F$ is the new feed tonnage in tons/h
- $E$ is the % screen efficiency (% undersize recovery)
- $u$ is the % undersize content in the crusher discharge
- $r$ is the % undersize content in the retained on the screen
- $t$ is the % undersize content in the new feed $F$,
  where undersize is the material finer than the aperture of the screen deck.

Previous works performed by various researchers (Testut, 1958; Magerowski and Karra, 1982; Stamboltzis, 1985) showed that:

$$R_g = \frac{W_g}{F} = \frac{100 - u}{u - r} \quad (1)$$
Another equation proposed earlier (Allis-Chalmers, 1953; Flavel, 1977) gives the circulating load ratio $R_g$ as a function of $e$ and $x$,

$$R_g = \frac{1}{e - 1} x$$  \hspace{1cm} (2)

where $e$ is the % efficiency of the oversize removal, $x = (100 - u)$ is the percent oversize in the crusher discharge, and $R_g$ is the circulating load ratio expressed as fraction of the new feed $F$. But, since $e = (100 - r)$, Eq. (2) can be shown to be identical with Eq. (1). The percent undersize recovery $E$ in the regular closed circuit, by definition, is given from:

$$E = \frac{F}{(F + W_g) \times \frac{u}{100} \times 100} = \frac{100^2}{1 + \left(\frac{W_g}{F}\right) \times u}$$  \hspace{1cm} (3)

Substituting for $W_g$ from Eq. (1) and rearranging gives:

$$\frac{100 - u}{u - r} = \frac{100 \times (100 - E)}{E \times r}$$  \hspace{1cm} (4)

Thus, from Eq. (1):

$$R_g = \frac{100 \times (100 - E)}{E \times r}$$  \hspace{1cm} (5)

It was also shown (Karra, 1979; Stamboltzis, 1985) that:

$$R_g = \frac{100^2}{E \times u} - 1$$  \hspace{1cm} (6)

from which:

$$R_g + 1 = \frac{100^2}{E \times u}$$  \hspace{1cm} (7)

Equation (7) can also be derived by substitution of $(u - r)$ from Eq. (1) into Eq. (3) and by suitable rearrangement.

b. Reverse closed circuit

In the reverse closed circuit (Fig. 2):

$R_v$ is the circulating load ratio expressed as fraction of the new feed $F$

$W_v$ is the circulating load tonnage in tons/h

$F$ is the new feed tonnage in tons/h

$E$ is the % screen efficiency (% undersize recovery)

$u$ is the % undersize content in the crusher discharge

$r$ is the % undersize content in the retained on the screen

$t$ is the % undersize content in the new feed $F$.

The circulating load ratio $R_v$ is given by Karra (1979):

$$R_v = \frac{1}{E \times u} \times \left(100^2 - E \times t\right)$$  \hspace{1cm} (8)
and by Stamboltzis (1985):

$$ R_s = \frac{W_r}{F} = \frac{100 - t}{u - r} \quad (9) $$

Fig. 2. Schematic representation of a reverse closed circuit along with relevant notation.

According to the definition of the screen efficiency or percent undersize recovery $E$, we have:

$$ E = \frac{F}{F \times \left( \frac{t}{100} \right) + W_r \times \frac{u}{100}} \times 100 = \frac{100^2}{t + \left( \frac{W_r}{F} \right) \times u} \quad (10) $$

Substituting for $\left( \frac{W_r}{F} \right)$ from Eq. (9) into Eq.(10) and rearranging yields:

$$ \frac{100 - E}{E} = \frac{r \times (100 - t)}{100 \times (u - r)} \quad (11) $$

Rearranging Eq. (11) and taking into account Eq. (9) yields:

$$ R_s = \frac{100 - E}{E} \times \frac{100}{r} \quad (12) $$

Equation (12) is identical to Eq.(5) derived earlier for the regular closed circuit.

The main effort, made before, aimed to the formation of suitable relationships, between the various parameters of the closed circuits, for the construction of nomographs. As it was shown previously, these relationships were derived by proper use or combination of the already known equations and those here proposed.
Study of Eqs. (2), (6) and (8)

a. Equation (2) is not valid when \[ \frac{u}{x} = \frac{100 - r}{100 - u} \leq 1. \]

If \[ \frac{100 - r}{100 - u} = 1, \] then \( u = r. \) This equality means that, the screen acts rather as a sample divider and not like a particle filter.

If \[ \frac{100 - r}{100 - u} < 1, \] then \( r > u. \)

The above inequality denotes that the screen increases the undersize content in its two products (passing 100% and retained \( r \)). The overall increase of the undersize content found in the screen products means that the screen behaves like a crusher, which is obviously impossible.

b. Equation (6) can be converted to:

\[
R_g = \frac{100}{u} \times \left( \frac{100 - u}{E - \frac{u}{100}} \right)
\]

(13)

The minimum value of \( R_g \) given from the above equation occurs when \( u = 100\% \) and is given by:

\[
R_{g,\text{min}} = \left( \frac{100}{E} - 1 \right)
\]

(14)

But, if \( u = 100\% \) (perfect operation of the crusher or reduction of the whole oversize fed to the crusher), then there is no need for a screen to control the crusher product. That means that the circuit is open \( (R_g = 0). \)

Equation (14) gives also the same result for \( E = 100\%. \)

c. Equation (8) can be also converted to Eq. (15):

\[
R_v = \frac{100}{u} \times \left( \frac{100 - t}{E - \frac{t}{100}} \right)
\]

(15)

If \( u = 100\% \) (perfect operation of the crusher), then \( R_v \) becomes minimum. The corresponding value is given by:

\[
R_{v,\text{min}} = \left( \frac{100}{E} - \frac{t}{100} \right)
\]

(16)

In that extreme case, the circulating load ratio \( R_v \) depends only on the screen efficiency \( E \), provided that the undersize content \( t \) in the feed remains constant (well homogenized new feed \( F \)).

Design of closed crushing circuits

Knowledge of the various equations governing the specific circuit (regular or reverse) is necessary for the design of closed crushing circuits.

a. Regular closed circuit

In most cases, for the calculation of the circulating load ratio \( R_g \), the acceptable range of values of some variables during operation is assumed. The primary variables, whose values can be more or less close to the
true and have a decisive role in the operation of the circuit, are \( u \) and \( E \) (e.g., 75-85\% for \( u \) and 85-95\% for \( E \)). There are also operational parameters of the equipment, e.g., crusher close side setting, vibration characteristics of the screen or feed rate etc., through which the performance of the circuit (\( E \) and \( u \)) is effectively controlled.

From Eq. (6) the circulating load ratio \( R_g \) can be found, provided that \( E \) and \( u \) are assumed. But, Eq. (6) can be converted to Eq. (7), which is in a form suitable for nomograph construction (Taggart, 1954; Behnke et al. 1966; Bass, 1968; Calculation & Shortcut Deskbook E/MJ, 1977). Taking logarithms of both sides and arranging properly the three axes results in a nomograph (Fig. 3).

![Nomograph](image)

Fig. 3. Nomograph (relationship between \( R_g, E \) and \( u \)) for the regular closed circuit.

**Example:** Calculate the circulating load ratio \( R_g \) in a regular closed circuit assuming that the screen efficiency \( E = 84\% \) and the percent undersize content in the crusher discharge \( u = 81\% \).

**Solution:**

1st step: From Eq. (6): \( R_g = \frac{100^3}{84 \times 81} - 1 = 0.4697 \)

2nd step: From Fig. 3: connect \( E = 84\% \) and \( u = 81\% \). The line between axes \( E \) and \( u \) intersects axis- \( R_g \) at 0.47.

The accuracy of the result predicted with the help of the nomograph, is significant.

**b. Reverse closed circuit**

For the reverse closed circuit, Karra (1979) was the first to derive Eq. (8), and a few years later followed Eq. (9) (Stamboltzis, 1985) and Eq. (12) (Tsakalakis, 1996). Equation (8) is a function of the three variables \( E, t, u \).
From Eqs. (9) and (12):

\[
R_v = \frac{100 - t}{u - r} = \frac{100 - E}{E} \times \frac{100}{r}
\]

from which

\[
\frac{100 - E}{E} \times \frac{100}{100 - t} = \frac{r}{u - r}
\]  \hspace{1cm} (17)

Setting

\[
\frac{100 - E}{E} \times \frac{100}{100 - t} = a = \text{constant}
\]  \hspace{1cm} (18)

Then, from Eqs. (17) and (18)

\[
\frac{r}{u - r} = a
\]  \hspace{1cm} (19)

Rearranging Eq. (19) yields:

\[
\frac{a + 1}{a} = \frac{u}{r}
\]  \hspace{1cm} (20)

From Eqs. (18), (19) and (12) nomographs are easily constructed. If, in Eq. (18) \(E\) is assumed and \(t\) is known, then \(\alpha\) can be calculated from the corresponding nomograph of Fig. 4.

![Fig. 4. Nomograph (relationship between \(E\), \(a\) and \(t\)) for the reverse closed circuit.](image)

From Eq. (20) and the values of \(\alpha\) (calculated above, Fig. 4) and \(u\) (assumed), \(r\) is calculated from the nomograph of Fig. 5.

With the help of Eq. (12), since \(E\) (assumed) and \(r\) (predicted from Fig. 5), \(R_v\) is obtained from the nomograph of Fig. 6.
For the calculation of $R_v$, three nomographs were used above.

However, the number of nomographs can be reduced by one with the procedure described below. Substitution of $r$ in Eq. (12) from Eq. (20) gives:
\[
R_v = 100 \times \frac{100 - E}{E} \times \frac{a + 1}{a} \times \frac{1}{u}
\]  

Equation (21) is an equation in proper form for the construction of a nomograph of the type shown in Fig. 7 (Taggart, 1954; Behnke et al., 1966; Bass, 1968).

![Fig. 7. Schematic representation of a four-variable nomograph.](image)

The nomograph of this type consists of five lines, three independent-variable lines (\(E, u, \alpha\)) and the dependent-variable line for \(R_v\). The fifth line is a reference line on the corresponding axis, line 1-2 intersects the reference line at point 0. The line from \(u\)-axis (point 3) passing through point 0 intersects axis-\(R_v\), at point 4, which gives the value of the circulating load \(R_v\).

The value of \(\alpha\) was obtained from the nomograph (Fig. 4) between \(E\) and \(t\). The actual four-variable nomograph constructed is that shown in Fig. 8.

![Fig. 8. Nomograph (relationship between \(R_v, E, \alpha, \) and \(u\)) for the reverse closed circuit.](image)
From the procedure described above it is concluded, that the calculation of the circulating load ratio $R_v$ can be performed with three different procedures:

1\textsuperscript{st}: the mathematical method of Eq. (8),
2\textsuperscript{nd}: the three-nomograph procedure (three nomographs of Figs. 4, 5 and 6) and
3\textsuperscript{rd}: the two-nomograph procedure of Figs. 4 and 8. This procedure does not require the calculation of the parameter $r$.

\textit{Example:} Calculate the circulating load ratio $R_v$ in a reverse closed circuit knowing that $t = 45\%$ and assuming that the screen efficiency $E = 86\%$ and the percent undersize content in the crusher discharge $u = 85\%$.

1\textsuperscript{st} solution: From Eq. (8) we obtain:

$$R_v = \frac{1}{E \times u} \times \left(100^2 - E \times t\right) = \frac{1}{86 \times 85} \times (100^2 - 86 \times 45) = 0.839$$

2\textsuperscript{nd} solution:

Since $E$, $t$ are known, from Eq. (18) and Fig. 4 the intermediate parameter $\alpha$ is calculated as $\alpha = 0.305$.

From Eq. (19) since $\alpha$ and $u$ are known, then from Fig. 5, $r = 19.7\%$.

From Eq. (12), since $E$ is assumed and $r$ was calculated, $R_v$ yields 0.82 (Fig. 6).

3\textsuperscript{rd} solution:

Since $E$, $t$ are known, from Eq. (18) and Fig. 4, $\alpha$ is calculated as $\alpha = 0.305$.

From Eq. (21) and from the known values of $E$, $u$ and $\alpha$, $R_v$ yields 0.82 (Fig. 8).

It should be pointed out that the use of the current nomographs can be seen as another attractive alternative to calculations performed by calculators and spreadsheets. The proposed procedure gives results comparable to those obtained from programmable calculators and spreadsheets. Besides that, it could be considered as a handy method for the designer, who would like to see visually the variation of the calculated parameters even for slight changes of the design parameters.

This technique could also be useful for the calculation of the screening efficiency (monitoring of the screen operation), if online measurement of the undersize percentages $t$, $u$ and $r$ was feasible.

\textbf{CONCLUSIONS}

From the above described procedure, it was shown that the closed circuits can be easily and accurately calculated and controlled by the exclusive use of nomographs. The procedure is simpler for the regular than for the reverse closed circuit, since the regular needs only one nomograph, while the reverse needs at least two for its calculation.

There are numerous combinations between the variables (mathematical expressions), which can be used for the construction of similar nomographs. The accuracy of the results always depends on the quality of the graphs, which here were drawn by hand. Thus the whole procedure can significantly simplify the calculations in designing closed crushing circuits.

Furthermore, it was shown that the circulating load ratios can be calculated in both types of circuits by the same equation (relationship between $R_g$, $R_v$, $E$ and $r$).

Here it must also be noted the mathematical consistency of Eq. (2) and the similarity of Eqs. (5), (12) and that of Eqs. (13) and (15) for the regular and the reverse closed circuits, respectively.

The present work helps the designing process engineer in calculations with closed circuits.
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